Nepal College of Information Technology

Balkumari Lalitpur

Assessment Exam Fall Semester 2008

Year: II Sem: I

Program: B.E/IT Time: 3hrs

FM: 100 PM: 50

***Probability & Queuing Theory***

**Attempt all questions.**

**Questions 1 to 6 carry 15 marks each.**

**Question 7 carries 10 marks.**

1. a. A firm has to choose seven people from its R and D team of ten to send to a conference on computer systems. How many ways are there of doing this

(i) when there are no restrictions,

(ii) when two of the team are so indispensable that only one of them can be permitted to go,

(iii) when it is essential that a certain member of the team goes?

b. A technician has to check the suitability of 3 solid crystal lasers and 2 carbon dioxide lasers for a given task. Using two coordinates so that (2, 1), for example, represents the event that the technician will find two of the solid crystal lasers and one of the carbon dioxide lasers suitable for the task, draw a diagram showing the 12 points of the sample space. If R is the event that equally many solid crystal lasers and carbon dioxide lasers are suitable for the task, T is the event that none of the carbon dioxide lasers is suitable for the task, and U is the event that fewer solid crystal lasers than carbon dioxide lasers are suitable for the task, express each of these events symbolically by listing its elements.

OR

1. a. We are given a box containing 5000 IC chips, of which 1000 are manufactured by company X and the rest by company Y. Ten percent of the chips made by company X and 5 percent of the chips made by company Y are defective. If a randomly chosen chip is found to be defective, find the probability that it came from company X.

b. A r.v. X takes values -3, -2, -1, 0, 1, 2, 3 such that

P(X = 0) = P(X < 0) = P(X > 0);

P(X = -3) = P(X = -2) = P(X = -1) and

P(X = 1) = P(X = 2) = P(X = 3).

Obtain the probability mass function and distribution function of X.

2. a. The number of defects (i) in a certain model of a TV is distributed as follows:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| i | 0 | 1 | 2 | 3 | 4 | 5 |
| pi | 0.60 | 0.22 | 0.10 | 0.05 | 0.02 | 0.01 |

What is the expectation and variance of the number of defects in a set?

b. Let X and Y be independent rvs, each having pdf f(x, y) = 4xy, 0 ≤ x ≤ 1, 0 ≤ y ≤ 1. Find E(X), E(Y), E(X + Y) and E(XY).

c. Let X and Y have the joint pdf f(x, y) = (x + y)/21, x = 1, 2, 3 and y = 1, 2. Find the conditional expectation and variance of X when Y = 2.

3. a. Suppose it is known that 10% of the IC chips in a certain lot are defective. If 8 chips are selected at random, what is the probability of obtaining k defective chips (k = 0, 1, 2, ..., 8)? Also obtain the mean and variance for the data.

b. At a busy traffic intersection the probability p of an individual car having an accident is very small, say p = 0.0001. Suppose during a particular time period 4 P.M. to 6 P.M., a large number of cars, say 1000 pass through the intersection. Use Poisson distribution to find the probability of two or more accidents occurring during that period. Also give the mean and the variance of the distribution.

4. a. Given that f(x) = θ e-θx for 0 < x < ∞ and f(x) = 0 elsewhere, where θ > 0 is a constant, verify if it is a probability density function. If so, find the expectation and the variance.

b. The diameters of bolts produced by a particular machine follow a normal distribution with mean 1.34 cm and standard deviation 0.04 cm. A bolt is rejected if its diameter is less than 1.24 cm or more than 1.40 cm. (a) Find the percentage of bolts, which are accepted. The setting of the machine is altered so that the mean diameter changes but the standard deviation remains the same. With the new setting, 3% of the bolts are rejected because they are too large in diameter. (b) Find the new mean diameter of the bolts produced by the machine. (c) Find the percentage of bolts, which are rejected because they are too small in diameter.

5. a. There are two food stores A and B in a certain area. An investigation of the preferences of customers revealed that with probability 0.15 a customer of store A one week would go over to store B next week and with probability 0.10, a customer of store B would go over to store A. Initially 60% of the customers were with store A and 40% were with store B. What would you expect to be the percentage of customers in the two stores after 4 weeks? After a sufficiently long time?

b. Define a M/M/1: (N/FCFS) queue and give its operating characteristics. The capacity of a communication line is 2000 bits per second. The line is used to transmit eight-bit characters, so the maximum rate is 250 characters per second. The application calls for traffic from many devices to be sent on the line with a total volume of 12000 characters per minute. Obtain the operating characteristics for the system.

6. a. Consider the problem of designing a system with two identical processors. We have two independent job streams with respective average arrival rates λ1 = 20 and λ2 = 15 per hour. The average service time for both job types is 1/μ = 2 min = 1/30 hours. Should we dedicate a processor per job stream, or should we pool the job streams and processors together?

b. In a factory cafeteria, the customers (employees) have to pass through three counters. The customers buy coupons at the first counter, select and collect the snacks at the second counter and collect tea at the third. The server at each counter takes on an average 1.5 minutes although the distribution of service time is approximately Poisson at an average rate of 6 per hour. Calculate

i. the average time a customer spends waiting in the cafeteria,

ii. the average time of getting the service, and

iii. the most probable time in getting the service.

7. Write short notes on:

a. Continuous random variable and its use in queuing theory

b. Conditional probability and Baye’s theorem

c. Expectation of a random variable.